MAT120: Lecture 2 Handout

Informal Logic

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1 Logical Statements

1.1 What is a Logical Statement?

Logical statements are full, grammatical sentences which can either be evaluated as “true” or “false”. Technically, we call this assignment an “evaluation” or “an assignment of a truth-value”.

In a very informal sense, it’s nice to interpret logical statements as the building blocks of arguments: we can imagine little building blocks of different shapes and sizes. These blocks all have either a “T” or an “F” written on them, representing their truth-value (i.e. whether or not they are True or False). We can imagine making arguments by placing these blocks together in a way that creates a structure that stands on its own.

1.2 What Makes a Good Argument?

Continuing with our analogy, there are two ways in which an argument can be bad: faulty logic and false premises, which we call “invalidity” and “unsoundness”, respectively.

(1) **Faulty logic.** In our analogy of building blocks, we can imagine placing the blocks in such a way that the structure is “ugly” and doesn’t stand up on its own. This represents the situation in which we take the premises of an argument, and arrange them in an illogical or incorrect way to get to our conclusion. Such a situation is called an “invalid argument”.
As an example of this, consider the argument: “The Earth is a planet. Therefore, any other planet is the Earth”. The logical structure here is clearly faulty: not only is the conclusion wrong, but in the argument we seem to be assuming something like “for all things P, if P is a planet then P is the Earth”. This is clearly wrong, and is an example of a misuse of logic.

(2) **False premises.** Recall that each block has a little “T” or “F” written on it. Even if we were to stack the blocks into a nice structure that stands up on its own, we can still create a bad argument if some of the little blocks have an “F” written on them. This represents the situation in which we take a collection of statements (i.e. premises), arrange them in a valid way, but it just so happens that one of the premises is false.
We call such a situation “valid, but not sound”, where here “sound” means that the premises are true in the first place. An example of a valid argument that is not sound would be:

> “All dogs are cats. I have a dog, therefore I also have a cat”.

Now technically, the logic here is actually ok: the argument is saying something like “all P are Q. I have a P, therefore I have a Q”. The reason that this argument is “wrong” is because it is not sound: the premise “all dogs are cats” is simply wrong, and therefore the conclusion is not something to take seriously.
It should be noted that the soundness of an argument does not require the conclusion to be obviously false: in principle, we could use an unsound argument and accidentally conclude something true. For example:

> “All dogs are cats. I don’t have a cat, therefore I don’t own a dog.”

As a matter of fact, I happen to not own a dog. So, if I were to make this argument, the conclusion of “I don’t own a dog” is technically true, despite getting there by an unsound argument.

(3) **A good argument.** So far, we have seen that "validity" deals with the argument's correct use of logic, and "soundness" deals with the argument's use of facts. A good and convincing argument requires both of these features: we need a good argument to be a series of correct uses of logic, starting from true premises.
In our block analogy: if we have all the little blocks and stack them up into a nice structure, and all of the blocks have a "T" written on them, then we have a good argument. So: a good argument is a mixture of good logic (i.e. stacking the blocks up in the right way) and good premises (i.e. the logical statements that we take as assumptions are all true). An example of a valid and sound argument might be:

> "All dogs are animals. I have a dog, therefore I have an animal".

This argument is so correct that it feels obvious – precisely because it is both valid and sound.

1.3 Examples of Logical Statements

Logical statements are sentences that have a definite truth-value. In fact, not every sentence we use in day-to-day life has this property. Here are some examples and non-examples of logical statements.

(1) "Some roses have thorns". This is an example of a logical statement (in fact, it is true).
(2) "There is an orange on my desk". This is an example of a logical statement (it is false).
(3) "I am richer than Shohei Ohtani". This is an example of a logical statement (it is false, sadly).
(4) "All roses are flowers". This is an example of a logical statement (it is true).
(5) "Please pass the water". This is not a logical statement, since it cannot be evaluated as either true or false. In fact, it is just a request.
(6) "Are you tired?" This is not a logical statement; it's a question. There is no "true" or "false" that we can understand here.
(7) "Gpauwibpuaregbiaebg". This is not a logical statement because it's simply meaningless.

Exercise

For the following sentences, determine whether they are logical statements or not.

(1) Please close the door.
(2) Is the homework due today?
(3) Tokyo is in Japan.
(4) 2 is greater than 3.
(5) The movie starts at 3pm.
(6) I am currently sitting down.
(7) John is tall.

Solutions

Items (3), (4), (5) and (6) are logical statements. Items (1) and (2) are not logical statements. As for item (7), it depends on who you ask! We will now discuss this in some detail.

1.4 A Note on Vagueness

Sometimes it's very unclear if something is a statement or not. Everyday language is a social practice, and interpersonal communication often involves a large number of non-verbal assumptions or conventions. Thus, the language of the real-world is often vague and it is not clear if a sentence actually has a truth-value or not. Let's again consider the sentence:

> "John is tall."

Although this might seem like there is a definite truth-value to this sentence, it is not always as easy as that. In fact, the word "tall" is a relative term, for instance John is tall compared to an insect but he is not tall compared to a tree. In isolation, the sentence "John is tall" does not actually specify what we should be comparing John to.

Of course, you may fight back at this stage and say "obviously the comparison is being made between John and other humans". However, that is simply an assumption that you made based on context, and in fact no such content can be found in the three-word sentence "John is tall." Put differently, it is you who expected the comparison to be against other humans, and that is merely an assumption based on context. A more pedantic, yet more correct, interpretation of the sentence is that "John is tall" is simply too vague to be considered a genuine logical statement – it cannot be definitely evaluated as either true or false without providing some more information.

The problem of vagueness is a rich philosophical subject, and it is rather interesting. On the one hand, we can agree that vague statements are neither true nor false. On the other hand, we can also observe that vagueness is how humans communicate. Statements like "birds fly" or "John is tall" are vague, but we all know what they mean.

Many philosophers and linguists have tried to tackle the problem of vagueness from a logical standpoint: questions like this may be considered an example of philosophical logic – the use of logical techniques in the analysis of philosophical problems. However, generally speaking, in this course we are going to live simply and pretend that the deep philosophical issues of vagueness don't exist for now.

1.5 The Hair Salon

We will now present another interesting logical puzzle, which looks clear and precise, yet doesn't have a definite truth-value.

Imagine a small village in the Japanese countryside. In this village there is one hair salon, and since the village is small, there is only one woman who works there. This woman has a rule for herself, which I will call (R):

> (R): "I will only cut the hair of women who do not cut their own hair".

This seems pretty reasonable, since women who cut their own hair don't have a need to go to the hair salon. However, we can get ourselves very confused if we ask the question: Does the woman cut her own hair?

There are two situations: (1) she cuts her own hair and (2) she does not. In situation (1), according to the rule (R) she cuts the hair of women who do not cut their own hair, so this implies that she does not cut her own hair! In situation (2) she does not cut her own hair, which means she can apply the rule and cut her own hair! This leads us back to situation (1) and we enter into an infinite loop of logic: (1) implies (2) implies (1) implies (2) and so on. But, (1) and (2) are opposite statements... so which one is true?

In this example we have stumbled across a strange “infinite loop” in logic, in which there is a precise (i.e. non-vague) sentence, yet its truth-value constantly switches between true and false whenever we try to consider it. Such a situation is called a “logical paradox”, and it is rather confusing. The resolution of paradoxes like this again falls within the academic field of philosophical logic.

As a matter of fact, our hair salon example is an adaptation of a very famous paradox known as the “Barber’s paradox”. A closely related paradox is the “liar’s paradox”, which revolves around the sentence “this sentence is a lie”. Imagine saying that sentence: if you say the sentence and it is true, then you are not lying, since your statement is true. However the sentence itself says that the sentence is a lie, so you are lying! Alternatively, if you say “this sentence is a lie” and that is false, that means that “this sentence is not a lie”, i.e. you are telling the truth! So, we keep flicking between true and false forever, in a similar sense that our unfortunate hair salon employee did.

It is worth mentioning that in this course we will not attempt to solve any of these deeper logical problems. Instead, to keep things simple I will only give you unambiguous, clear examples of logical statements.

2 Quantifiers

2.1 A Thought Experiment

I would like you to imagine God, who is all-powerful. In front of God is a cardboard box. God takes all the objects in the universe and lines them all up in front of him. And then, one by one, he goes through the line and places all of the guitars in the cardboard box. At the end of this procedure, God shuts the cardboard box and places a sticker on the outside of the box which reads “guitars”. He then takes that box and places it on a pile with all the other cardboard boxes of stuff (there is a box for “pickles”, and a box for “paper”, and a box for “numbers”, and so on).

As well, I’d like you to pretend that the same object can go in multiple possible cardboard boxes, for instance I might go in the box labelled “British people”, but I’d also go in the box labelled “Davids” and I’d also go in the box labelled “Human Beings”.

Why am I telling you this? Because, in some (much more limited, finite) sense, we humans are always putting objects into the boxes of our minds. This is a process called cognitive categorization.

2.2 Cognitive Categorization

We humans are constantly identifying objects and categorizing them according to the judgements we make. This is often an instant, unconscious process, and it is a fundamental function of sufficiently advanced life forms. Other animals (dogs, cats, monkeys) also always do this: they build some internal theories of the world by making judgements about the things that they see. For us humans, whenever we meet a new object, we immediately “put it in a box” in our minds and assign some linguistic label to it.

2.3 Quantifiers

Now, quantifiers are small parts of logical statements that deal with categorization. They are features of language that require us to go into the boxes and check if things have a particular way of being. Examples of logical statements that include quantifiers are:

(1) All guitars are musical instruments.
(2) There is a blue guitar.
(3) There are no guitars made of water.

In each of these statements, we are somehow assessing “the box of all guitars” in order to determine the truth-value of the statement. In the first case, we have to check every single guitar and see if it’s a musical instrument. In the second case, we have to try and find an example of a blue guitar within the box. In the third case, we have to check every single guitar and make sure that it’s not made of water.

In normal language, quantifiers are words like “all, some, there is, there are none, every, no” and so on. They are small parts of sentences that force us to check whether some objects in a category have a particular property (like being made of water, for example).

2.4 Exercise (Identifying Quantifiers)

Exercise

Identify the quantifiers in the following statements.

(1) Every human being has a unique set of fingerprints.
(2) There are no states of the U.S. that start with a Q.
(3) No students in this class have visited Antarctica.
(4) In China, there are 18 cities with a population over 10 million.
(5) At least two people in this room have an Android smartphone.

Solutions

(1) “every”. (2) “there are no”. (3) “no”. (4) “there are”. (5) “at least two”.

2.5 More Ways to Express Quantifiers

In English, many different words can actually be used to express the same quantifier. Here are some examples of synonyms of quantifiers.

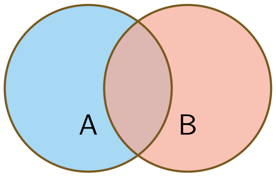
(1) All, every, every single
(2) Some, a few
(3) There is, there exists
(4) No, there aren’t, there are no, none of

As a matter of fact, there are actually only two real logical quantifiers: “all” and “there is”. The rest of the quantifiers can be built from these two.

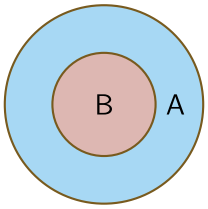
2.6 Alternate Representation: Venn Diagrams

Another way to represent statements involving quantifiers is to use Venn diagrams. Instead of using cardboard boxes, these are diagrams that are written down in terms of circles. We represent a category of objects (guitars, things made of water, blue things, etc.) using a circle, where anything inside the circle is inside the category, and anything outside the circle is not inside the category. For example, you can imagine this like drawing a ring on the ground, and putting all the guitars inside the ring and leaving all the not-guitars outside the ring.

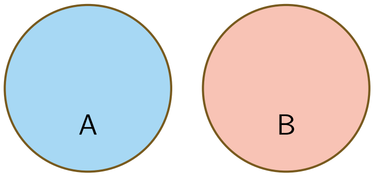
We can represent statements involving quantifiers by using these circles. Suppose that we have two circles, one circle represents all the objects A, and the other circle represents all the objects B. There are three possibilities for how these two circles interact with each other:



(a) Some B's are also A's



(b) All B's are also A's



(c) No B's are also A's

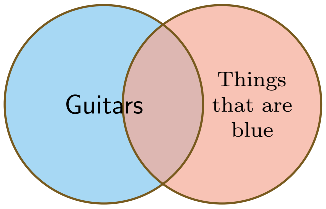
(a) The two circles overlap a little bit. In this case, there are some things that are both A and B at the same time. This represents the statement "some A's are also B's".

(b) One of the circles is smaller and fully contained in the other one, for example B's circle is totally contained within A's circle. In this case, this represents the statement "all B's are also A's".

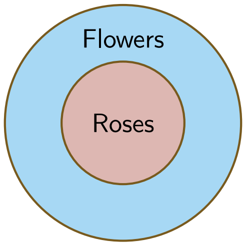
(c) Both of the circles are totally separated from each other. In this case, this represents the statement "no B's are also A's and no A's are also B's".

2.7 Example of Venn Diagrams

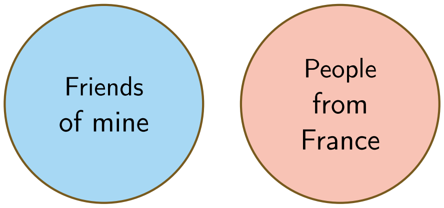
As an example of this, here are 3 Venn diagrams:



(a) "Some guitars are blue"



(b) "All roses are flowers"



(c) "I am not friends with any French people"

(1) Two circles that overlap a little bit. If I let category A = guitars and category B = blue things, then two overlapping circles represent the statement "there are some blue guitars".

(2) Two circles where one is contained in the other. If I now let category A = roses and category B = flowers, then the circle for A being contained in the circle for B represents the statement "all roses are flowers".

(3) Two circles that are totally separated. If I let category A = friends of mine and category B = French people, then the two circles being separated represent the statement "I am not friends with any French people".

3 Connectives

3.1 What is a Connective?

A connective is a way of taking two logical statements and combining them together to create a new logical statement, i.e. we "connect" two statements together somehow. Since the new thing is also a logical statement, it has a truth-value (it is either true or false), and the exact nature of this truth-value will depend on the way in which we connected the starting statements together. This all sounds a little bit confusing, but in fact it gets quite easy once we see some examples of connectives – in fact, we use connectives all the time in daily life without even realizing it. Put differently, logical statements and logical connectives are so obvious and so common in day-to-day life that we take them for granted.

We will now study 5 basic ways to combine logical statements into new ones. These are fundamental, and any other clever combination of logical statements can always be described in terms of these five fundamental techniques (in fact, secretly you only need two connectives). The five types of connectives are: Negation, Conjunction, Disjunction, Implication and Equivalence. In common English, it is better to understand these fancy words as:

(1) Negation = Not
(2) Conjunction = And
(3) Disjunction = Or
(4) Implication = If … then …
(5) Equivalence = … if and only if …

3.2 Negation

The easiest way to think of negation is that it means "not" in everyday language. Negation takes a single logical statement and forms a new statement meaning "it is not the case that …". If the original statement is true, the negated statement is false, and if the original statement is false, then the negated statement is true. So, negation simply flips the truth-value of the statement you start with.

For example: the negation of the statement "it is raining outside" is "it is not raining outside".

Another way to think of negation is like an instant refutation of the previous statement. If I said "I made a million dollars this morning before my breakfast", the obvious response would be "no you didn't". The latter statement is a negation: you are really saying "no, you did not make a million dollars this morning before breakfast". This is all negation is: the negation of a statement is the statement with the opposite truth-value.

3.3 Conjunction

The easiest way to think of conjunction is that it means "and" in everyday language. A conjunction combines two logical statements into one larger statement meaning "Statement 1 and Statement 2". The conjunction is only true when both individual statements are true. So, that means a conjunction is false if one of the two statements is false (or both are false). Why? Because conjunction simply means "and", which itself means "both at the same time".

For example: if Statement 1 is “I drank coffee this morning” and Statement 2 is “I ate oatmeal this morning”, then the conjunction of these two statements is “I drank coffee and ate oatmeal this morning”. The conjunction can only possibly be true if indeed I did both things. If it were the case that I didn’t drink coffee (or didn’t have oatmeal), then the conjunction “I drank coffee and ate oatmeal this morning” would be false.

Just in case you are curious: the word “conjunction” sounds very similar to the word “conjoined”, which is a term commonly used for the very rare situation in which twin babies are joined together in the womb. Here, the prefix “con” means “with” or “together”, so “conjoined” simply means “joined together”. That’s all a conjunction is: we take two logical statements and we evaluate their truth-values “joined together”.

3.4 Disjunction

The easiest way to think of disjunction is that it means “or” in everyday language. A disjunction combines two logical statements into one larger statement meaning “Statement 1 or Statement 2.” In logic, the word “or” is usually inclusive, meaning that it is ok for both to be true. In other words, the disjunction is true when at least one of the statements is true (including the case where both are true). Alternatively, the disjunction is false only when both statements are false.

For example: if Statement 1 is “John is in a cafe” and Statement 2 is “John is in Tokyo”, then the disjunction of these two statements is “John is in a cafe or he is in Tokyo”. The disjunction is true in 3 situations: John is in a cafe outside of Tokyo, John is in Tokyo but not in a cafe, or both at the same time, i.e. John is in a cafe in Tokyo somewhere. This statement can only be false if John is neither in a cafe nor in Tokyo.

The prefix “dis” means “apart, or separated”, for instance the word “discard” means “to throw away a card”, and has its origin in old fashioned card games. In most card games, you carry many cards in your hands – the cards that are under your direct control. Some games require you to “throw away” some card from your hand, that is, you “discard” by separating a card from the rest of those in your hand. In this sense, “dis” means “apart from”. Coming back to the word “disjunction”, we now see that it means something like “joined, but apart”. In fact, that perfectly describes the way to find the truth-value of a disjunction: we consider the two truth-values of the starting statements separately, and either one of them being true will cause the disjunction to be true.

3.5 Exercise (Negation, conjunction, disjunction)

Exercise

Before getting to the other connectives, we will have a quick exercise. Consider now two statements. Statement 1 says “John is on the moon” and Statement 2 says “John is in Tokyo”.

(1) Negate Statement 1.
(2) Negate Statement 2.
(3) Form a conjunction of Statement 1 and your answer to (1). Is your new statement true or false?
(4) Form a disjunction of Statement 2 and your answer to (2). Is your new statement true or false?

Solutions

(1) John is not currently on the moon.
(2) John is not currently in Tokyo.
(3) John is currently on the moon and John is currently not on the moon. (False)
(4) John is currently in Tokyo or he is currently not in Tokyo. (True)

3.6 Implication

The best way to think about implication is that it's an "if… then…" statement in everyday language. An implication combines two logical statements into one larger statement meaning "If Statement 1 is true, then Statement 2 is true". The implication is false only in one case: when Statement 1 is true but Statement 2 is false. In every other case, the implication is true.

A good way to think about it is that an implication is a conditional rule (or a promise) that is broken exactly when the first part happens but the second part does not. Imagining something like airport security may work here: when going through the security gates, the officers tell you "if you have a laptop then take it out of the bag and put it in a separate tray", only then, they let you pass. This is an implication: it's a rule that you may need to apply, depending on whether you have a laptop or not. The only situation in which the officer won't let you through the gate is if you violate the rule, i.e. you have a laptop in your bag and you don't take it out.

For another example: Let Statement 1 be "you are over 20 years old" and Statement 2 be "you can buy a beer at the konbini". Then the implication formed from Statement 1 to Statement 2 will be "if you are over 20 years old then you can buy a beer at the konbini". This statement will only be false if Statement 1 were true but Statement 2 were false, i.e. you are over 20 years old, but for some reason you can still not buy a beer at the konbini.

3.7 Equivalence

The best way to think about an equivalence is that it means "if and only if" in everyday language. An equivalence combines two logical statements into one larger statement meaning "Statement 1 is true exactly when Statement 2 is true". The equivalence is true when the two statements have the same truth-value (i.e. they're both true or they're both false). The equivalence is false when they differ (one true and the other false). Equivalence expresses a two-way connection: each statement guarantees the other.

For example: if Statement 1 is "I like you" and Statement 2 is "you are not from France" then the equivalence formed from these two statements is "I like you if and only if you are not from France". This statement would be false if there were situations in which I liked a French person, or there were a non-French person that I didn't like.

3.8 Exercise (Implication and equivalence)

Exercise

Consider the two statements from the previous exercise, i.e. Statement 1 is “John is currently on the moon” and Statement 2 is “John is currently in Tokyo”.

(1) Form the statement that expresses an implication from Statement 1 to the negation of Statement 2. Is this true or false?

(2) Form the statement that expresses an equivalence between Statement 1 and Statement 2.

Solutions

(1) If John is currently on the moon then John is currently not in Tokyo. (This is true.)

(2) John is currently on the moon if and only if he is currently in Tokyo.