MAT220 Lecture 15: Hypothesis Testing the Mean

Contents

In Lecture 14, we introduced the general idea of *hypothesis testing*. We learned how to state a null hypothesis and an alternate hypothesis, how to interpret a P-value, and how to compare that P-value with a significance level. In this lecture, we will focus on the probability calculation at the centre of the test. We will study three important situations:

1. testing the population mean μ when σ is known,
2. testing the population mean μ when σ is unknown, and
3. testing the binomial success probability p.

As we will see, these three situations follow the same general pattern, albeit with a different probability calculation at their cores.

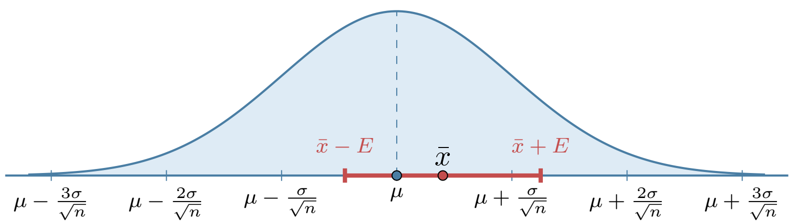
1 A Review of Hypothesis Testing

1.1 Estimation and hypothesis testing

When estimating an unknown population parameter, we take a sample and construct an interval of reasonable values around an estimate of it. For example, when estimating a population mean, we might construct a confidence interval centred at the observed sample mean:

$$[\bar{x} - E,\ \bar{x} + E]\,.$$

Here, the true population mean μ is unknown. The idea is that the size of the interval is calculated so that the method captures μ a chosen percentage of the time: if we repeatedly took samples of size n and computed all of the confidence intervals around their sample means, then a certain proportion of these intervals would contain the actual μ. Overall, this construction uses the Central Limit Theorem, which says that the sample mean $\bar{X}$ admits its own probability distribution with mean μ and variance $\frac{\sigma^2}{n}$.[1] The $\bar{x}$ and its confidence interval then live in this distribution:



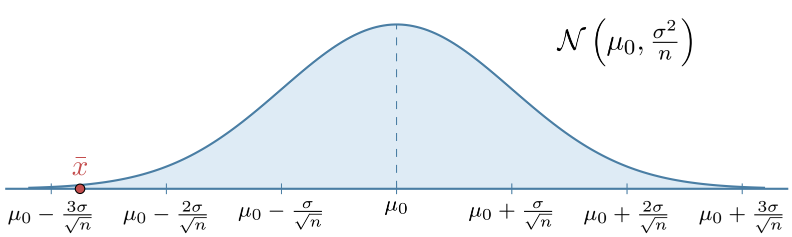
When performing hypothesis tests, we completely reverse our thought process. We begin by *assuming* a particular value of the population parameter and then ask whether the observed sample data would be reasonable under that assumption. For example, suppose that the null hypothesis claims

$$H_0 : \mu = \mu_0.$$

We *assume* that the real population mean is actually μ_0, and then explore the consequences of this assumption. If H_0 were true, then the Central Limit Theorem says that the sampling distribution should be:

$$\bar{X} \approx \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right).$$

Therefore, any observed sample mean $\bar{x}$ should appear somewhere within this distribution. If we take a sample of real-world data, and the sample mean $\bar{x}$ lands extremely far into a tail, then the data is unlikely under H_0:



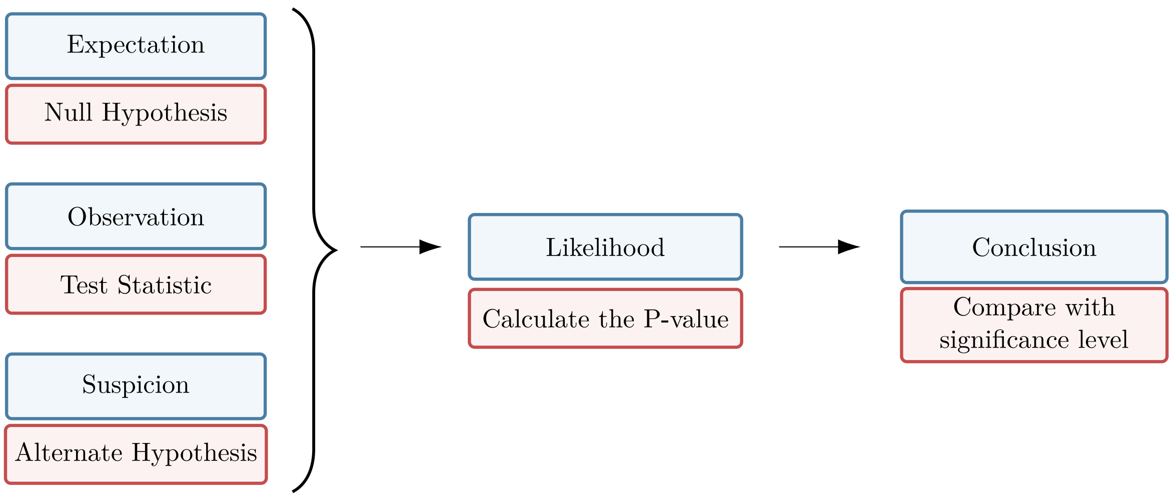
[1] Technically, n should be sufficiently large unless the population is normally distributed. Usually, we take samples of size 30 or larger.

Although these two methods appear to be very similar, there are key differences:

- in **estimation**, the population parameter is unknown and we construct an interval of plausible values;
- in **hypothesis testing**, we assume a particular value of the population parameter and ask whether the observed data is plausible under that assumption.

1.2 The general layout of a statistical test

Recall the general layout of a statistical test from Lecture 14:



The first stage involves reading and interpreting the situation. We identify:

- the claim being tested;
- the suspicion against that claim;
- the relevant sample information.

The second stage involves the main probability calculation. We then take this value (called the P-value), and make a judgement: we compare the resulting P-value with the chosen significance level and decide whether the sample provides enough evidence to reject the null hypothesis.

Important Definitions

- **Null hypothesis (H_0):** the statement that we want to test.
- **Alternate hypothesis (H_1):** the competing statement that reflects our suspicion.
- **Test statistic:** the numerical information obtained from the sample data.
- **P-value:** the probability of obtaining the observed test statistic, or something more extreme, assuming that H_0 is true.
- **Significance level (α):** the cut-off probability that tells us what counts as unacceptably unlikely.

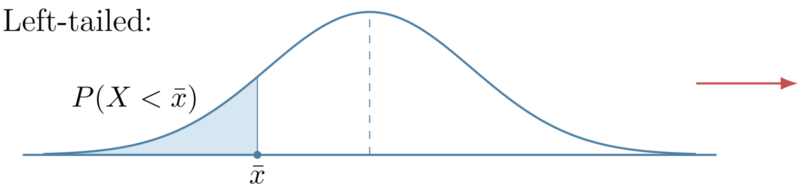
1.3 Different tails, different P-values

It is common for students to think that the P-value somehow tells us how likely the null hypothesis is. In fact, this is **not** true – instead, the P-value measures how unusual the observed sample (or something worse) would be *if* the null hypothesis were true. The alternate hypothesis tells us which outcomes count as "worse" than the observed sample data. There are three common forms of alternate hypotheses:

$$H_1 : \begin{cases} \text{true value} < \text{claimed value}, & \text{left-tailed test}, \\ \text{true value} > \text{claimed value}, & \text{right-tailed test}, \\ \text{true value} \neq \text{claimed value}, & \text{two-tailed test}. \end{cases}$$

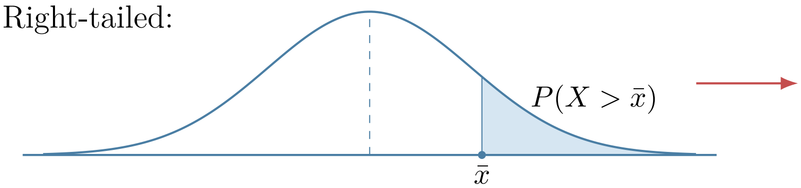
According to the statement of the alternate hypothesis, the calculation of the P-value will change depending on which tail we are considering:

Left-tailed:



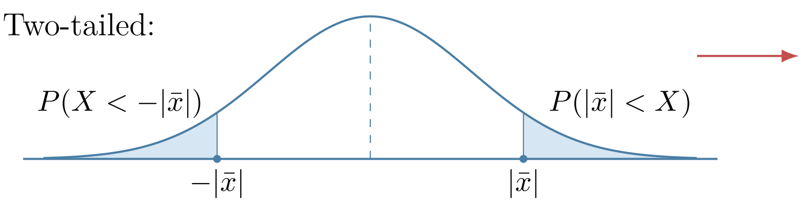
P-value $= P(X < \bar{x})$

Right-tailed:



P-value $= P(X > \bar{x})$

Two-tailed:



$$\begin{aligned} \text{P-value} &= P(X < -|\bar{x}|) + P(X > |\bar{x}|) \\ &= 2P(X > |\bar{x}|) \end{aligned}$$

2 Testing μ with Known σ

2.1 The general idea

Suppose that we want to test a claim about the population mean μ, and suppose that the population standard deviation σ happens to be known. In this situation, the null hypothesis will take the form:

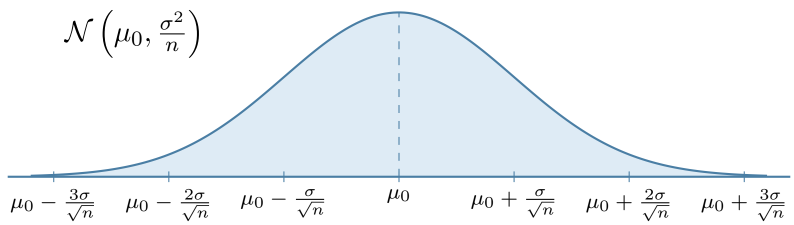
$$H_0 : \mu = \mu_0,$$

where here μ_0 is the claimed value of μ. Sample data will typically be provided, meaning that we have a definite value of the sample size n and the sample mean $\bar{x}$.

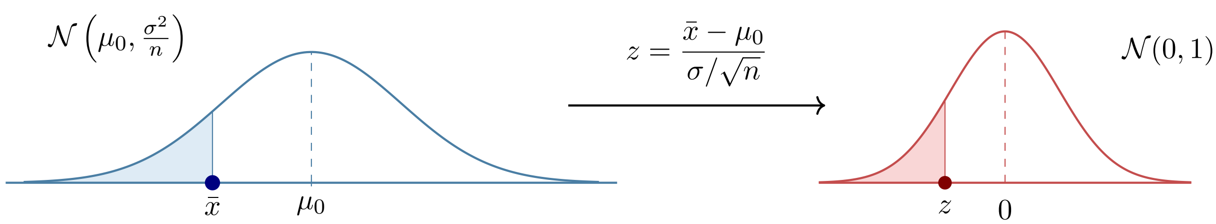
In order to calculate the relevant P-value, we assume that the null hypothesis is true, that is, we assume that the population mean is actually μ_0. Under this assumption, the Central Limit Theorem tells us that the sampling distribution for the sample means will have the parameters:

$$\bar{X} \approx \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right),$$

meaning that the centre of this normal distribution is the claimed mean μ_0, and the standard error is $\frac{\sigma}{\sqrt{n}}$.



If we wanted to calculate the probability of a given $\bar{x}$ (or worse), then we would need to convert the area into an area of the standard normal distribution. The connection between the two distributions is the familiar z-score formula. For example, for a left-tailed test, the P-value associated with an observation $\bar{x}$ might be an area like this:



Since the centre of our sampling distribution is assumed to be μ_0 and the standard error is $\frac{\sigma}{\sqrt{n}}$, we can use the formula:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$

The exact calculation of the P-value will then depend on what type of test we are performing. According to our discussion in Section 1, we either need to calculate:

- $P\left(Z < \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right)$ if it is a left-tailed test,
- $P\left(Z > \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right) = 1 - P\left(Z < \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right)$ if it is a right-tailed test, or perhaps
- $2P\left(Z > \left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right|\right)$ if it is a two-tailed test.

Testing μ with known σ

When the population standard deviation σ is known, use the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$

The probability of the resulting value can be calculated using the z-table for the standard normal distribution $Z \sim \mathcal{N}(0, 1)$.

2.2 Example 1: Coffee bags

Suppose that you work at a company that produces bags of coffee beans. The bags are supposed to have an average weight of 150 g, but you suspect that the true average is actually lower than this.

To test your suspicion, you take a simple random sample of 30 bags and find a sample mean of $\bar{x} = 145$ g. From historical data, you know that the population standard deviation happens to be $\sigma = 15$ g. We will now perform the test using a significance level of $\alpha = 0.01$.

Step 1: Identifying the information

According to our setup, the important values are:

$$\mu_0 = 150, \qquad n = 30, \qquad \bar{x} = 145, \qquad \sigma = 15, \qquad \alpha = 0.01.$$

The company claims that the population mean is 150 g, while we suspect that it is less than 150 g. Therefore the null and alternate hypotheses are:

$$H_0 : \mu = 150, \qquad \text{and} \qquad H_1 : \mu < 150,$$

respectively. Since the alternate hypothesis is a "less than" statement, we are performing a left-tailed test.

Step 2: Calculate the P-value

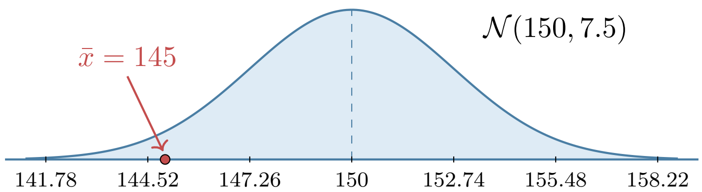
If we assume that the true mean is 150 g, the Central Limit Theorem tells us that the sampling distribution is approximately normal, with parameters:

$$\bar{X} \approx \mathcal{N}\left(150, \frac{15^2}{30}\right) = \mathcal{N}\left(150, 7.5\right).$$

Here, the standard error is

$$\frac{\sigma}{\sqrt{n}} = \sqrt{7.5} \approx 2.74.$$

Therefore, the horizontal axis of the null sampling distribution may be labelled approximately as follows:



We now calculate the z-score:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{145 - 150}{15/\sqrt{30}} \approx -1.83.$$

Therefore, we can compute the probability of $\bar{X}$ being less than 145 g by finding the probability of Z being less than -1.83. Using a z-table, we have:

$$P(\bar{X} < 145) = P(Z < -1.83) \approx 0.0336.$$

Since we are performing a left-tailed test, this is our P-value. In other words, if the true average weight were really 150 g, there would be about a 3.36% chance of obtaining a sample mean of 145 g or less from a sample of this size.

Step 3: Compare the P-value with α

A direct comparison between the P-value and α gives:

$$0.0336 > 0.01.$$

Therefore, at a significance level of 1%, we do not reject H_0.

Step 4: Give an interpretation

One example interpretation could be as follows.

Interpretation

At the 1% significance level, our sample does not provide convincing evidence that the true average weight of the coffee bags is less than 150 g.

3 Testing μ with Unknown σ

3.1 Where the previous method breaks

Suppose again that we want to test

$$H_0 : \mu = \mu_0,$$

but this time the population standard deviation σ is unknown. In the previous section, we used the z-score formula

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

to calculate the appropriate P-value. However, if σ is unknown then we can't actually compute this number. Instead, we have to proceed as in Lecture 13 and estimate σ with the *sample* standard deviation s. We can simply replace σ with s in the formula above to obtain a different quantity:

$$\frac{\bar{x}-\mu_0}{s/\sqrt{n}}.$$

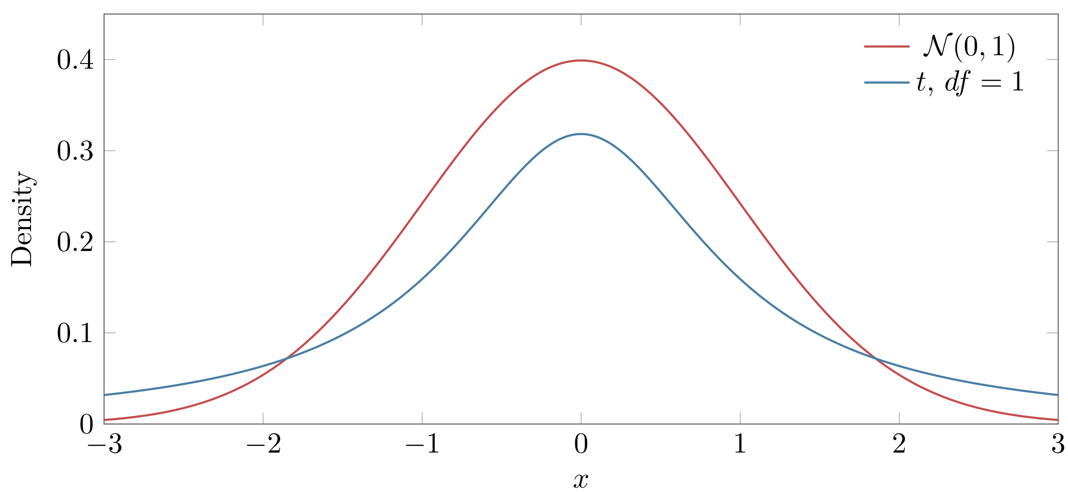
However, as we saw in Lecture 13, this quantity is no longer normally distributed. Instead, we have introduced an extra layer of uncertainty when estimating σ, and therefore we need to use a Student's t-distribution. We will now briefly review this.

3.2 A review of Student's t-distribution

Assume that the original population is normally distributed with mean μ. For a sample of size n, with sample mean $\bar{X}$ and sample standard deviation S (now viewed as a random variable in its own right), the quantity

$$T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$$

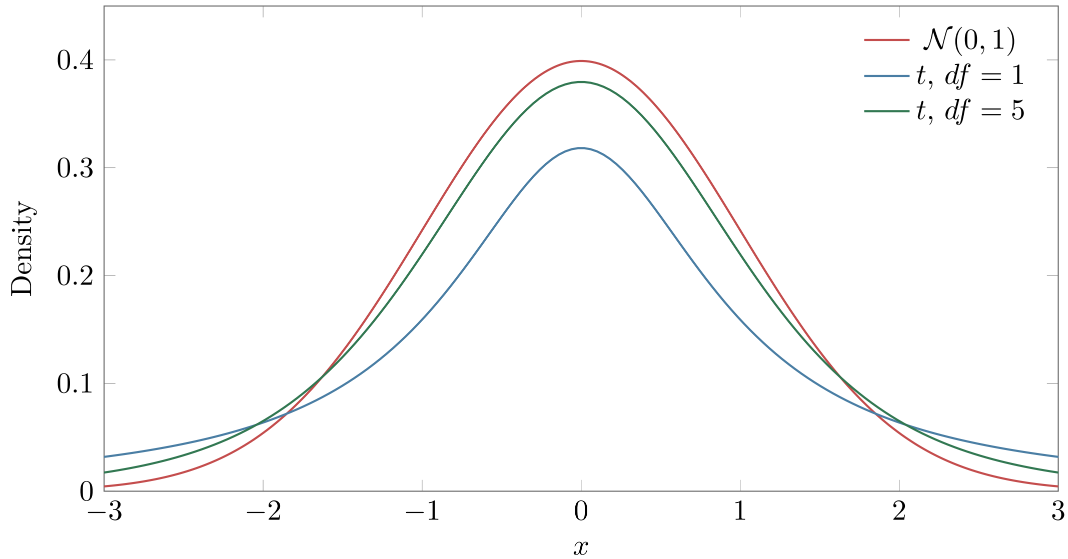
has Student's t-distribution with $df = n-1$. Student's t-distributions somewhat resemble the standard normal distribution, except that they have heavier tails:



In particular, these distributions still satisfy lots of the same properties as the standard normal distribution:

- they are still symmetric about 0,
- the total area under the curve is still 1, and
- probabilities still correspond to areas under the curve.

However, there are also some important differences. Firstly, there is more than one t-distribution. Instead, there is an entire *family* of these curves, depending on the different values of df. The idea here is that as we increase the size of our sample, the initial random noise in observations should ultimately cancel out, giving us a more accurate estimate of the true value σ. As expected, when we increase the value of n, the value of s tends to get closer to σ, and therefore the t-distribution looks more and more like $\mathcal{N}(0,1)$:



3.3 Testing μ with unknown σ

Instead of using the z-score formula from Section 2, in the absence of σ we need to use the t-statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}},$$

where again we use μ_0 as the assumed value of the population mean μ. This formula allows us to calculate the P-value associated with a sample observation.

Testing μ with unknown σ

When the population standard deviation σ is unknown, use

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

The resulting value allows us to compute the P-value of an observed sample by computing probabilities under the Student's t-distribution with $df = n - 1$.

Again, the exact P-value will depend on what the alternate hypothesis says to do:

$$\begin{aligned} H_1 : \mu < \mu_0 \quad &\implies \quad \text{P-value} = P(T < t), \\ H_1 : \mu > \mu_0 \quad &\implies \quad \text{P-value} = P(T > t), \\ H_1 : \mu \neq \mu_0 \quad &\implies \quad \text{P-value} = 2P(T > |t|). \end{aligned}$$

Of course, the obvious question at this stage is: how do we actually compute these probabilities in the first place? In the past, if we wanted to compute the value of a probability in the normal distribution then we would compute a z-score and then read the probability $P(Z < z)$ from a table. Similarly, there is a table associated with the Student's t-distributions. In the Appendix below, you will see such a table. The idea is essentially the same as the z-table from Lecture 10: we compute a value of t, and then read the associated probability $P(T < t)$ from the table. Whenever we want a probability such as $P(T > t)$ or $P(t_1 < T < t_2)$, we can use the same tricks as in the normal distribution:

How to find other areas in a t-distribution

- **Areas to the left of t:** read off $P(T < t)$ directly.
- **Areas to the right of t:** use $P(T > t) = 1 - P(T < t)$.
- **Areas between two values:** use $P(t_1 < T < t_2) = P(T < t_2) - P(T < t_1)$.
- **Negative values:** by symmetry, for $t > 0$,

$$P(T < -t) = 1 - P(T < t).$$

The only major difference in the tables is the precision. In the z-table, we can look up z-values to two decimal places, but in the t-table, we can only look up t-values to one decimal place. The other information in the table provides answers for the different values of df, up to 30.

Exercise

Using the supplied left-tail t-table in the Appendix, find:

(a) $P(T < 1.3)$ when $df = 18$;

(b) $P(T < -2.0)$ when $df = 7$;

(c) $P(T > 1.8)$ when $df = 12$.

Solution

(a)

$$P(T < 1.3) \approx 0.8950.$$

(b)

$$P(T < -2.0) \approx 0.0428.$$

(c) From the table,

$$P(T < 1.8) \approx 0.9515.$$

Therefore,

$$P(T > 1.8) = 1 - P(T < 1.8) \approx 1 - 0.9515 = 0.0485.$$

3.4 Example 2: Underweight onigiri

Suppose that every day you buy the same advertised 200 g onigiri from a convenience store. However, you suspect that the company is misleading its customers, and that the true average weight is actually lower than 200 g.

To test your suspicion, you weigh one randomly selected onigiri on each of the 31 days in July. From this sample, you calculate a sample mean of $\bar{x} = 197$ g and a sample standard deviation of $s = 9$ g.

We will now perform the test using a significance level of $\alpha = 0.05$.

Step 1: Identifying the information

According to our setup, the important values are:

$$\mu_0 = 200, \qquad n = 31, \qquad \bar{x} = 197, \qquad s = 9, \qquad \alpha = 0.05.$$

The company claims that the population mean is 200 g, while we suspect that it is less than 200 g. Therefore the null and alternate hypotheses are:

$$H_0 : \mu = 200, \qquad \text{and} \qquad H_1 : \mu < 200,$$

respectively. Since the alternate hypothesis is a "less than" statement, we are performing a left-tailed test.

Step 2: Calculate the P-value

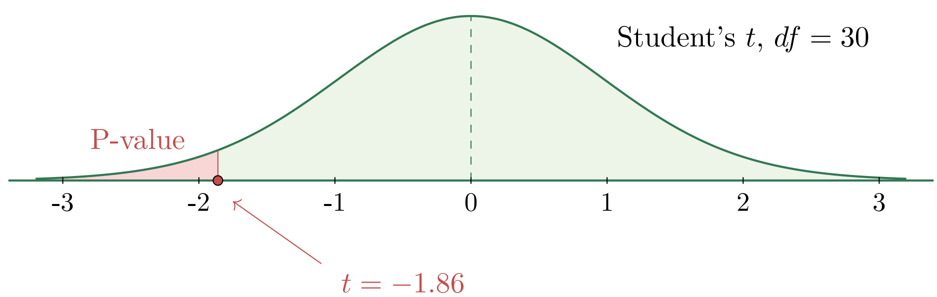
In the previous example, we knew the true population standard deviation σ, and therefore we were able to use the standard normal distribution. In this example, however, we only know the sample standard deviation $s = 9$. Therefore, we need to calculate a t-statistic instead:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{197 - 200}{9/\sqrt{31}} \approx -1.86.$$

Since the sample size is $n = 31$, the number of degrees of freedom is:

$$df = n - 1 = 31 - 1 = 30.$$

Therefore, assuming that the null hypothesis is true, the calculated value $t = -1.86$ lives in a Student's t-distribution with $df = 30$:



Since we are performing a left-tailed test, the P-value is the probability of obtaining a t-statistic of -1.86 or less:

$$\text{P-value} = P(T < -1.86), \qquad df = 30.$$

Our t-table only lists values to one decimal place, so -1.86 lies between the listed values -1.9 and -1.8. From the table, we have:

$$P(T < -1.9) \approx 0.03354$$

and

$$P(T < -1.8) \approx 0.04096.$$

Since -1.86 lies between these two values, we estimate the P-value by taking their midpoint:

$$\text{P-value} \approx \frac{0.03354 + 0.04096}{2} = 0.03725.$$

In other words, if the true average weight were really 200 g, there would be about a 3.7% chance of obtaining a t-statistic of -1.86 or less from a sample of this size.

Step 3: Compare the P-value with α

A direct comparison between the P-value and α gives:

$$0.03725 < 0.05.$$

Therefore, at a significance level of 5%, we reject H_0.

Step 4: Give an interpretation

Interpretation

At the 5% significance level, our sample provides convincing evidence that the true average weight of the onigiri is less than the advertised 200 g.

4 Testing the Binomial Success Probability p

4.1 The sampling distribution of the sample proportion

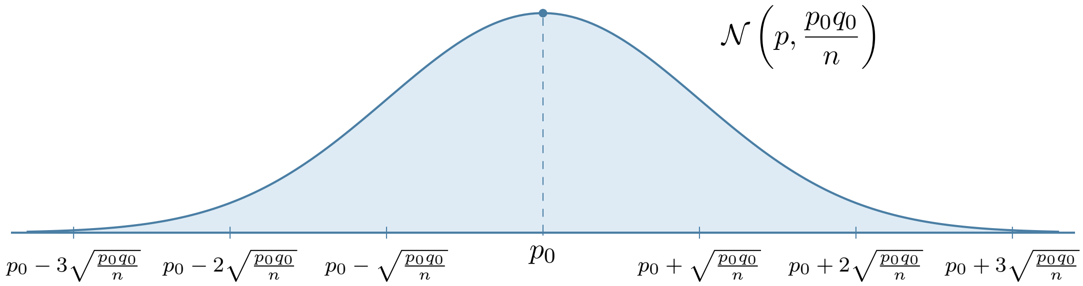
Suppose now that we are working with a binomial experiment with an unknown success probability p. As a sample, we observe n trials of the experiment, and we witness r successes and $n - r$ failures. From this, we can test a claim about the true probability p, that is, a null hypothesis of the form:

$$H_0 : p = p_0.$$

Using the sample data, we can calculate the sample proportion $\hat{p} = \frac{r}{n}$. Assuming that the null hypothesis is true, the normal approximation to this binomial experiment will be the sampling distribution of $\hat{P}$, arranged as a normal distribution with parameters

$$\hat{P} \approx \mathcal{N}\left(p_0, \frac{p_0 q_0}{n}\right),$$

where here $q_0 = 1 - p_0$. Geometrically, the centre of this distribution is the claimed value p_0, and its standard error is the term $\sqrt{\frac{p_0 q_0}{n}}$, meaning that our normal distribution counts out in terms of the latter quantity:



Therefore, the corresponding z-score is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}.$$

Testing the binomial success probability p

When testing the true value of a binomial success probability p, use the test statistic

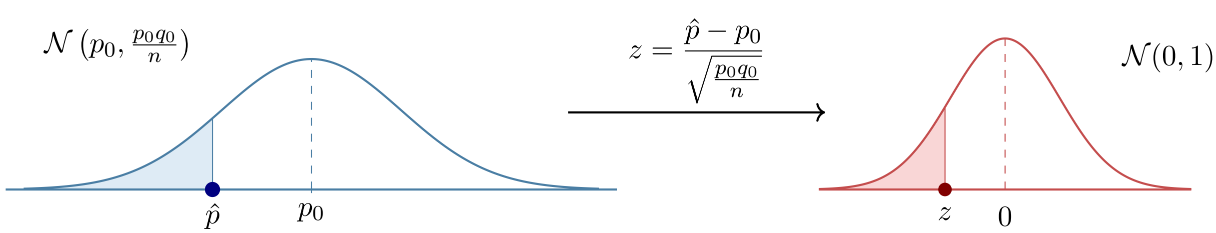
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}},$$

where here

$$\hat{p} = \frac{r}{n} \qquad \text{and} \qquad q_0 = 1 - p_0.$$

The probability of the resulting value can be calculated using the z-table for the standard normal distribution $Z \sim \mathcal{N}(0, 1)$.

As with Section 2, geometrically the formula transforms our probability calculation into an area of the normal distribution, which we can determine from the z-table. The formula in the above box provides the link to the standard normal distribution, so that areas can be calculated easily:



Before applying the approximation, we should always check that $np_0 \geq 5$ and $nq_0 \geq 5$ – this suggests that the normal approximation is reasonable. Also, notice that for a hypothesis test the standard error is calculated using the null value p_0:

$$\sqrt{\frac{p_0 q_0}{n}}.$$

This differs from a confidence interval for p, where we used the observed values $\hat{p}$ and $\hat{q}$:

$$\sqrt{\frac{\hat{p}\hat{q}}{n}}.$$

4.2 Example 3: Penalty kicks

Suppose that your mate John claims that he is as good as Lionel Messi at taking penalty kicks. Messi's career penalty success rate is 78%. So, you watch John practice, and amazingly he scores 52 penalties out of 55 attempts. After witnessing John's greatness, you suspect that his true accuracy may be even better than Lionel Messi's.

We will now perform the hypothesis test using a significance level of $\alpha = 0.01$.

Step 1: Identifying the information

According to our setup, the important values are:

$$p_0 = 0.78, \qquad q_0 = 1 - p_0 = 0.22, \qquad n = 55, \qquad r = 52, \qquad \alpha = 0.01.$$

John's initial claim corresponds to a success probability of 0.78, while we suspect that his true probability is greater than 0.78. Therefore the null and alternate hypotheses are:

$$H_0 : p = 0.78, \qquad \text{and} \qquad H_1 : p > 0.78,$$

respectively. Since the alternate hypothesis is a "greater than" statement, we are now performing a *right*-tailed test.

Step 2: Calculate the P-value

Before using the normal approximation, we should first check the approximation conditions. We have:

$$np_0 = 55(0.78) = 42.9 \qquad \text{and} \qquad nq_0 = 55(0.22) = 12.1.$$

Both of these values are greater than 5, so indeed a normal approximation is reasonable.

To calculate the P-value, we first need to find the observed sample proportion:

$$\hat{p} = \frac{r}{n} = \frac{52}{55} \approx 0.945.$$

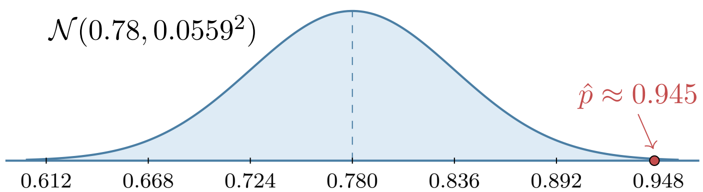
If we assume that the true success probability is $p_0 = 0.78$, then the normal approximation tells us that the sampling distribution of $\hat{P}$ is approximately normal, with parameters:

$$\hat{P} \approx \mathcal{N}\left(0.78, \frac{(0.78)(0.22)}{55}\right).$$

In particular, this distribution is centred at our claimed p_0. Here, the standard error is:

$$\sqrt{\frac{p_0 q_0}{n}} = \sqrt{\frac{(0.78)(0.22)}{55}} \approx 0.0559.$$

Therefore, the horizontal axis of the sampling distribution may be labelled approximately as follows:



Just by inspecting the graph, we see that $\hat{p}$ is extremely far from the center, and therefore extremely unlikely. Nonetheless, we may determine this probability by calculating the relevant z-score:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.945 - 0.78}{\sqrt{\frac{(0.78)(0.22)}{55}}} \approx 2.96.$$

Therefore, we can compute the probability of $\hat{P}$ being greater than approximately 0.945 by finding the probability of Z being greater than 2.96. Using a z-table, we first find that:

$$P(Z < 2.96) = 0.99846.$$

This value is the probability to the *left* of 2.96. We are performing a right-tailed test, so we would like to find the area to the right instead. We have:

$$P(Z > 2.96) = 1 - P(Z < 2.96) = 1 - 0.99846 = 0.00154.$$

This is our P-value. In other words, if John's true probability of scoring a penalty kick were really 0.78, there would be about a 0.154% chance of observing a sample proportion of approximately 0.945 or greater from 55 attempts.

Step 3: Compare the P-value with α

A direct comparison between the P-value and α gives:

$$0.00154 < 0.01.$$

Therefore, at a significance level of 1%, we reject H_0.

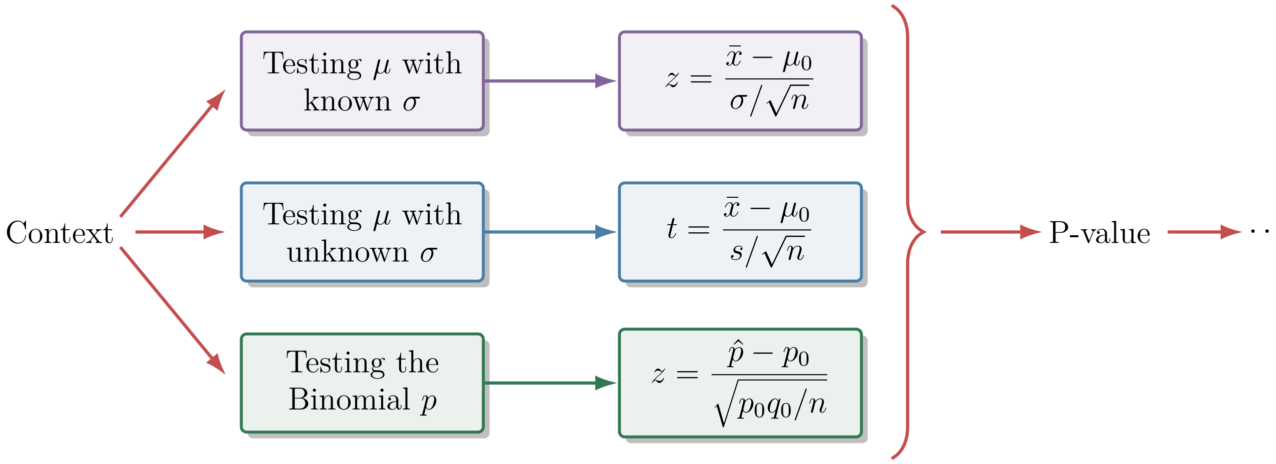
Step 4: Give an interpretation

Interpretation

At the 1% significance level, John's results provide convincing evidence that his true probability of scoring a penalty kick is greater than 0.78.

5 Summary

Throughout this lecture, we have seen three unique settings in which we can perform a hypothesis test on a population parameter. Generally speaking, it is our job to correctly identify the context of any given problem, and then to know which test statistic to compute in each situation. Roughly, we have the following flow:



Regardless of the context, the overall workflow of a hypothesis test still remains the same:

1. we state H_0 and H_1, and record some sample data, then
2. we calculate the appropriate test statistic and P-value, then
3. we compare the P-value with α and decide whether to reject H_0, and finally
4. we give a conclusion in the context of the problem.

6 Appendix: Tables for Student's t-Distributions

The tables below provide approximate answers to the area under students t-distributions at various degrees of freedom. In distinction to the z-table we have seen previously, here the tables are only accurate to one decimal place. In the event that t-statistics are obtained to two decimal places, there are three options:

- if the second decimal is a $1, 2$ or 3, then we may round *down* to the first decimal place,
- if the second decimal is a $7, 8$ or 9, then we may round *up* to the first decimal place, and
- if the second decimal is a $4, 5$ or 6, then we may take the midpoint of the two nearest answers on the table.

Replacing the second decimal are rows that correspond to areas under the curve for values of *df* ranging from 1 up to 30.

Table 1: The Left-tailed Probabilities $P(T < t)$ for positive t, with df = 1–15

t	**df 1**	**df 2**	**df 3**	**df 4**	**df 5**	**df 6**	**df 7**	**df 8**	**df 9**	**df 10**	**df 11**	**df 12**	**df 13**	**df 14**	**df 15**
0.0	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
0.1	0.53173	0.53527	0.53667	0.53742	0.53788	0.53820	0.53843	0.53860	0.53873	0.53884	0.53893	0.53900	0.53906	0.53912	0.53917
0.2	0.56283	0.57001	0.57286	0.57438	0.57532	0.57596	0.57642	0.57676	0.57704	0.57726	0.57743	0.57759	0.57771	0.57782	0.57792
0.3	0.59277	0.60376	0.60812	0.61044	0.61188	0.61285	0.61355	0.61409	0.61450	0.61484	0.61511	0.61534	0.61554	0.61571	0.61585
0.4	0.62112	0.63608	0.64203	0.64520	0.64716	0.64850	0.64946	0.65019	0.65076	0.65122	0.65159	0.65191	0.65217	0.65240	0.65260
0.5	0.64758	0.66667	0.67428	0.67834	0.68085	0.68256	0.68380	0.68473	0.68546	0.68605	0.68654	0.68694	0.68728	0.68758	0.68783
0.6	0.67202	0.69528	0.70460	0.70958	0.71267	0.71477	0.71629	0.71744	0.71835	0.71907	0.71967	0.72017	0.72059	0.72095	0.72127
0.7	0.69440	0.72180	0.73284	0.73875	0.74243	0.74493	0.74674	0.74811	0.74919	0.75006	0.75077	0.75136	0.75187	0.75230	0.75268
0.8	0.71478	0.74618	0.75890	0.76574	0.76999	0.77289	0.77500	0.77659	0.77784	0.77885	0.77968	0.78037	0.78096	0.78146	0.78190
0.9	0.73326	0.76845	0.78277	0.79050	0.79531	0.79860	0.80099	0.80280	0.80422	0.80536	0.80630	0.80709	0.80776	0.80833	0.80883
1.0	0.75000	0.78868	0.80450	0.81305	0.81839	0.82204	0.82469	0.82670	0.82828	0.82955	0.83060	0.83148	0.83222	0.83286	0.83341
1.1	0.76515	0.80698	0.82416	0.83346	0.83927	0.84325	0.84614	0.84834	0.85006	0.85145	0.85259	0.85355	0.85436	0.85506	0.85566
1.2	0.77886	0.82350	0.84187	0.85182	0.85805	0.86232	0.86541	0.86777	0.86961	0.87110	0.87233	0.87335	0.87422	0.87497	0.87563
1.3	0.79129	0.83838	0.85777	0.86827	0.87485	0.87935	0.88262	0.88510	0.88705	0.88862	0.88991	0.89099	0.89191	0.89270	0.89339
1.4	0.80257	0.85176	0.87200	0.88295	0.88980	0.89448	0.89788	0.90046	0.90249	0.90412	0.90546	0.90658	0.90754	0.90836	0.90907
1.5	0.81283	0.86380	0.88471	0.89600	0.90305	0.90786	0.91135	0.91400	0.91607	0.91775	0.91912	0.92027	0.92125	0.92209	0.92282
1.6	0.82219	0.87463	0.89605	0.90758	0.91475	0.91964	0.92318	0.92587	0.92797	0.92966	0.93105	0.93221	0.93320	0.93404	0.93478
1.7	0.83075	0.88438	0.90615	0.91782	0.92506	0.92998	0.93354	0.93622	0.93833	0.94002	0.94140	0.94256	0.94354	0.94439	0.94512
1.8	0.83859	0.89317	0.91516	0.92688	0.93412	0.93902	0.94256	0.94522	0.94730	0.94897	0.95034	0.95148	0.95245	0.95328	0.95400
1.9	0.84579	0.90109	0.92318	0.93488	0.94207	0.94692	0.95040	0.95302	0.95506	0.95669	0.95802	0.95914	0.96008	0.96089	0.96158
2.0	0.85242	0.90825	0.93034	0.94194	0.94903	0.95379	0.95719	0.95974	0.96172	0.96331	0.96460	0.96567	0.96658	0.96736	0.96803
2.1	0.85854	0.91473	0.93672	0.94817	0.95512	0.95976	0.96306	0.96553	0.96744	0.96896	0.97020	0.97123	0.97209	0.97283	0.97347
2.2	0.86420	0.92060	0.94241	0.95367	0.96045	0.96495	0.96813	0.97050	0.97233	0.97378	0.97496	0.97593	0.97675	0.97745	0.97805
2.3	0.86945	0.92593	0.94751	0.95853	0.96511	0.96945	0.97250	0.97476	0.97650	0.97787	0.97898	0.97990	0.98067	0.98132	0.98189
2.4	0.87433	0.93077	0.95206	0.96282	0.96919	0.97335	0.97627	0.97841	0.98005	0.98134	0.98238	0.98324	0.98396	0.98457	0.98509
2.5	0.87888	0.93519	0.95615	0.96662	0.97275	0.97674	0.97950	0.98153	0.98307	0.98428	0.98525	0.98604	0.98671	0.98727	0.98775
2.6	0.88312	0.93923	0.95981	0.96998	0.97588	0.97967	0.98229	0.98419	0.98563	0.98675	0.98765	0.98839	0.98900	0.98951	0.98995
2.7	0.88709	0.94292	0.96311	0.97295	0.97861	0.98221	0.98468	0.98646	0.98780	0.98884	0.98967	0.99035	0.99090	0.99137	0.99177
2.8	0.89081	0.94630	0.96607	0.97559	0.98100	0.98442	0.98674	0.98840	0.98964	0.99060	0.99136	0.99198	0.99249	0.99291	0.99327
2.9	0.89430	0.94941	0.96875	0.97794	0.98310	0.98633	0.98851	0.99005	0.99120	0.99208	0.99278	0.99334	0.99380	0.99418	0.99450
3.0	0.89758	0.95227	0.97117	0.98003	0.98495	0.98800	0.99003	0.99146	0.99252	0.99333	0.99396	0.99447	0.99488	0.99522	0.99551
3.1	0.90067	0.95490	0.97335	0.98189	0.98657	0.98944	0.99134	0.99267	0.99364	0.99437	0.99495	0.99541	0.99578	0.99608	0.99634
3.2	0.90359	0.95733	0.97533	0.98355	0.98800	0.99070	0.99247	0.99369	0.99458	0.99525	0.99577	0.99618	0.99652	0.99679	0.99702
3.3	0.90634	0.95958	0.97713	0.98503	0.98926	0.99180	0.99344	0.99457	0.99539	0.99599	0.99646	0.99683	0.99713	0.99737	0.99757
3.4	0.90895	0.96166	0.97877	0.98636	0.99037	0.99275	0.99428	0.99532	0.99606	0.99661	0.99704	0.99737	0.99763	0.99784	0.99802
3.5	0.91141	0.96359	0.98026	0.98755	0.99136	0.99359	0.99500	0.99596	0.99664	0.99714	0.99751	0.99781	0.99804	0.99823	0.99839
3.6	0.91375	0.96538	0.98162	0.98862	0.99223	0.99432	0.99563	0.99651	0.99713	0.99758	0.99792	0.99818	0.99838	0.99855	0.99869
3.7	0.91598	0.96705	0.98286	0.98958	0.99300	0.99496	0.99617	0.99698	0.99754	0.99795	0.99825	0.99848	0.99866	0.99881	0.99893
3.8	0.91809	0.96860	0.98400	0.99045	0.99369	0.99552	0.99664	0.99738	0.99789	0.99826	0.99853	0.99874	0.99890	0.99902	0.99913
3.9	0.92010	0.97005	0.98504	0.99123	0.99430	0.99601	0.99705	0.99773	0.99819	0.99852	0.99876	0.99894	0.99909	0.99920	0.99929
4.0	0.92202	0.97140	0.98600	0.99193	0.99484	0.99644	0.99741	0.99803	0.99844	0.99874	0.99896	0.99912	0.99924	0.99934	0.99942

Table 2: The Left-tailed Probabilities $P(T < t)$ for positive t, with df=16–30

t	**df 16**	**df 17**	**df 18**	**df 19**	**df 20**	**df 21**	**df 22**	**df 23**	**df 24**	**df 25**	**df 26**	**df 27**	**df 28**	**df 29**	**df 30**
0.0	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
0.1	0.53921	0.53924	0.53928	0.53930	0.53933	0.53935	0.53937	0.53939	0.53941	0.53943	0.53944	0.53946	0.53947	0.53948	0.53950
0.2	0.57800	0.57807	0.57814	0.57820	0.57825	0.57830	0.57834	0.57838	0.57842	0.57845	0.57848	0.57851	0.57854	0.57856	0.57858
0.3	0.61598	0.61609	0.61619	0.61628	0.61636	0.61644	0.61650	0.61656	0.61662	0.61667	0.61672	0.61676	0.61680	0.61684	0.61688
0.4	0.65278	0.65293	0.65307	0.65319	0.65330	0.65340	0.65349	0.65358	0.65365	0.65372	0.65379	0.65385	0.65390	0.65396	0.65400
0.5	0.68806	0.68826	0.68843	0.68859	0.68873	0.68886	0.68898	0.68909	0.68919	0.68928	0.68936	0.68944	0.68951	0.68958	0.68964
0.6	0.72155	0.72179	0.72201	0.72220	0.72238	0.72254	0.72268	0.72281	0.72294	0.72305	0.72315	0.72325	0.72333	0.72342	0.72349
0.7	0.75301	0.75330	0.75356	0.75379	0.75400	0.75420	0.75437	0.75453	0.75467	0.75480	0.75493	0.75504	0.75515	0.75525	0.75534
0.8	0.78229	0.78263	0.78293	0.78320	0.78344	0.78367	0.78387	0.78405	0.78422	0.78438	0.78452	0.78465	0.78478	0.78489	0.78500
0.9	0.80927	0.80965	0.81000	0.81031	0.81059	0.81084	0.81107	0.81128	0.81147	0.81165	0.81181	0.81196	0.81210	0.81223	0.81236
1.0	0.83390	0.83433	0.83472	0.83506	0.83537	0.83565	0.83591	0.83614	0.83636	0.83655	0.83674	0.83691	0.83706	0.83721	0.83735
1.1	0.85620	0.85667	0.85709	0.85746	0.85780	0.85811	0.85839	0.85864	0.85888	0.85909	0.85929	0.85948	0.85965	0.85981	0.85996
1.2	0.87620	0.87670	0.87715	0.87756	0.87792	0.87825	0.87855	0.87882	0.87907	0.87931	0.87952	0.87972	0.87990	0.88007	0.88023
1.3	0.89399	0.89452	0.89500	0.89542	0.89581	0.89616	0.89647	0.89676	0.89703	0.89727	0.89750	0.89770	0.89790	0.89808	0.89825
1.4	0.90970	0.91025	0.91074	0.91118	0.91158	0.91194	0.91227	0.91257	0.91285	0.91310	0.91333	0.91355	0.91375	0.91394	0.91411
1.5	0.92346	0.92402	0.92452	0.92498	0.92538	0.92575	0.92609	0.92639	0.92667	0.92693	0.92717	0.92739	0.92760	0.92779	0.92797
1.6	0.93542	0.93599	0.93650	0.93695	0.93736	0.93773	0.93807	0.93838	0.93866	0.93892	0.93916	0.93938	0.93959	0.93978	0.93996
1.7	0.94576	0.94632	0.94683	0.94728	0.94768	0.94805	0.94839	0.94869	0.94897	0.94923	0.94947	0.94969	0.94989	0.95008	0.95026
1.8	0.95463	0.95518	0.95568	0.95612	0.95652	0.95688	0.95720	0.95750	0.95778	0.95803	0.95826	0.95848	0.95868	0.95886	0.95904
1.9	0.96220	0.96273	0.96321	0.96364	0.96403	0.96437	0.96469	0.96498	0.96524	0.96549	0.96571	0.96592	0.96611	0.96629	0.96646
2.0	0.96861	0.96913	0.96959	0.97000	0.97037	0.97070	0.97100	0.97128	0.97153	0.97176	0.97198	0.97217	0.97236	0.97253	0.97269
2.1	0.97403	0.97452	0.97495	0.97534	0.97569	0.97601	0.97629	0.97655	0.97679	0.97701	0.97721	0.97740	0.97757	0.97773	0.97788
2.2	0.97858	0.97904	0.97945	0.97981	0.98014	0.98043	0.98070	0.98094	0.98116	0.98137	0.98155	0.98173	0.98189	0.98204	0.98218
2.3	0.98238	0.98281	0.98319	0.98352	0.98383	0.98410	0.98435	0.98457	0.98478	0.98496	0.98514	0.98530	0.98544	0.98558	0.98571
2.4	0.98554	0.98594	0.98629	0.98660	0.98688	0.98713	0.98735	0.98756	0.98775	0.98792	0.98807	0.98822	0.98836	0.98848	0.98860
2.5	0.98816	0.98853	0.98885	0.98913	0.98938	0.98961	0.98982	0.99000	0.99017	0.99033	0.99047	0.99060	0.99072	0.99084	0.99094
2.6	0.99033	0.99066	0.99095	0.99121	0.99144	0.99164	0.99183	0.99200	0.99215	0.99229	0.99242	0.99253	0.99264	0.99274	0.99284
2.7	0.99211	0.99241	0.99267	0.99291	0.99311	0.99330	0.99346	0.99361	0.99375	0.99387	0.99398	0.99409	0.99419	0.99427	0.99436
2.8	0.99358	0.99385	0.99408	0.99429	0.99447	0.99464	0.99478	0.99491	0.99504	0.99515	0.99525	0.99534	0.99542	0.99550	0.99557
2.9	0.99478	0.99502	0.99523	0.99541	0.99557	0.99572	0.99585	0.99596	0.99607	0.99617	0.99625	0.99633	0.99641	0.99648	0.99654
3.0	0.99576	0.99597	0.99616	0.99632	0.99646	0.99659	0.99670	0.99680	0.99690	0.99698	0.99706	0.99713	0.99719	0.99725	0.99731
3.1	0.99656	0.99675	0.99691	0.99705	0.99718	0.99729	0.99739	0.99748	0.99756	0.99763	0.99769	0.99775	0.99781	0.99786	0.99791
3.2	0.99721	0.99738	0.99752	0.99764	0.99775	0.99785	0.99793	0.99801	0.99808	0.99814	0.99820	0.99825	0.99830	0.99834	0.99838
3.3	0.99774	0.99788	0.99801	0.99812	0.99821	0.99830	0.99837	0.99843	0.99849	0.99855	0.99860	0.99864	0.99868	0.99872	0.99875
3.4	0.99817	0.99830	0.99840	0.99850	0.99858	0.99865	0.99871	0.99877	0.99882	0.99887	0.99891	0.99894	0.99898	0.99901	0.99904
3.5	0.99852	0.99863	0.99872	0.99880	0.99887	0.99893	0.99899	0.99904	0.99908	0.99912	0.99915	0.99918	0.99921	0.99924	0.99926
3.6	0.99880	0.99890	0.99898	0.99905	0.99911	0.99916	0.99920	0.99924	0.99928	0.99931	0.99934	0.99937	0.99939	0.99941	0.99943
3.7	0.99903	0.99911	0.99918	0.99924	0.99929	0.99934	0.99937	0.99941	0.99944	0.99947	0.99949	0.99951	0.99953	0.99955	0.99957
3.8	0.99921	0.99928	0.99934	0.99940	0.99944	0.99948	0.99951	0.99954	0.99956	0.99959	0.99961	0.99963	0.99964	0.99966	0.99967
3.9	0.99936	0.99942	0.99948	0.99952	0.99956	0.99959	0.99962	0.99964	0.99966	0.99968	0.99970	0.99971	0.99973	0.99974	0.99975
4.0	0.99948	0.99954	0.99958	0.99962	0.99965	0.99968	0.99970	0.99972	0.99974	0.99975	0.99977	0.99978	0.99979	0.99980	0.99981

Table 3: The Left-tailed Probabilities $P(T < t)$ for negative t, with df=1–15

t	**df 1**	**df 2**	**df 3**	**df 4**	**df 5**	**df 6**	**df 7**	**df 8**	**df 9**	**df 10**	**df 11**	**df 12**	**df 13**	**df 14**	**df 15**
-4.0	0.07798	0.02860	0.01400	0.00807	0.00516	0.00356	0.00259	0.00197	0.00156	0.00126	0.00104	0.00088	0.00076	0.00066	0.00058
-3.9	0.07990	0.02995	0.01496	0.00877	0.00570	0.00399	0.00295	0.00227	0.00181	0.00148	0.00124	0.00106	0.00091	0.00080	0.00071
-3.8	0.08191	0.03140	0.01600	0.00955	0.00631	0.00448	0.00336	0.00262	0.00211	0.00174	0.00147	0.00126	0.00110	0.00098	0.00087
-3.7	0.08402	0.03295	0.01714	0.01042	0.00700	0.00504	0.00383	0.00302	0.00246	0.00205	0.00175	0.00152	0.00134	0.00119	0.00107
-3.6	0.08625	0.03462	0.01838	0.01138	0.00777	0.00568	0.00437	0.00349	0.00287	0.00242	0.00208	0.00182	0.00162	0.00145	0.00131
-3.5	0.08859	0.03641	0.01974	0.01245	0.00864	0.00641	0.00500	0.00404	0.00336	0.00286	0.00249	0.00219	0.00196	0.00177	0.00161
-3.4	0.09105	0.03834	0.02123	0.01364	0.00963	0.00725	0.00572	0.00468	0.00394	0.00339	0.00296	0.00263	0.00237	0.00216	0.00198
-3.3	0.09366	0.04042	0.02287	0.01497	0.01074	0.00820	0.00656	0.00543	0.00461	0.00401	0.00354	0.00317	0.00287	0.00263	0.00243
-3.2	0.09641	0.04267	0.02467	0.01645	0.01200	0.00930	0.00753	0.00631	0.00542	0.00475	0.00423	0.00382	0.00348	0.00321	0.00298
-3.1	0.09933	0.04510	0.02665	0.01811	0.01343	0.01056	0.00866	0.00733	0.00636	0.00563	0.00505	0.00459	0.00422	0.00392	0.00366
-3.0	0.10242	0.04773	0.02883	0.01997	0.01505	0.01200	0.00997	0.00854	0.00748	0.00667	0.00604	0.00553	0.00512	0.00478	0.00449
-2.9	0.10570	0.05059	0.03125	0.02206	0.01690	0.01367	0.01149	0.00995	0.00880	0.00792	0.00722	0.00666	0.00620	0.00582	0.00550
-2.8	0.10919	0.05370	0.03393	0.02441	0.01900	0.01558	0.01326	0.01160	0.01036	0.00940	0.00864	0.00802	0.00751	0.00709	0.00673
-2.7	0.11291	0.05708	0.03689	0.02705	0.02139	0.01779	0.01532	0.01354	0.01220	0.01116	0.01033	0.00965	0.00910	0.00863	0.00823
-2.6	0.11688	0.06077	0.04019	0.03002	0.02412	0.02033	0.01771	0.01581	0.01437	0.01325	0.01235	0.01161	0.01100	0.01049	0.01005
-2.5	0.12112	0.06481	0.04385	0.03338	0.02725	0.02326	0.02050	0.01847	0.01693	0.01572	0.01475	0.01396	0.01329	0.01273	0.01225
-2.4	0.12567	0.06923	0.04794	0.03718	0.03081	0.02665	0.02373	0.02159	0.01995	0.01866	0.01762	0.01676	0.01604	0.01543	0.01491
-2.3	0.13055	0.07407	0.05249	0.04147	0.03489	0.03055	0.02750	0.02524	0.02350	0.02213	0.02102	0.02010	0.01933	0.01868	0.01811
-2.2	0.13580	0.07940	0.05759	0.04633	0.03955	0.03505	0.03187	0.02950	0.02767	0.02622	0.02504	0.02407	0.02325	0.02255	0.02195
-2.1	0.14146	0.08527	0.06328	0.05183	0.04488	0.04024	0.03694	0.03447	0.03256	0.03104	0.02980	0.02877	0.02791	0.02717	0.02653
-2.0	0.14758	0.09175	0.06966	0.05806	0.05097	0.04621	0.04281	0.04026	0.03828	0.03669	0.03540	0.03433	0.03342	0.03264	0.03197
-1.9	0.15421	0.09891	0.07682	0.06512	0.05793	0.05308	0.04960	0.04698	0.04494	0.04331	0.04198	0.04086	0.03992	0.03911	0.03842
-1.8	0.16141	0.10683	0.08484	0.07312	0.06588	0.06098	0.05744	0.05478	0.05270	0.05103	0.04966	0.04852	0.04755	0.04672	0.04600
-1.7	0.16925	0.11562	0.09385	0.08218	0.07494	0.07002	0.06646	0.06378	0.06167	0.05998	0.05860	0.05744	0.05646	0.05561	0.05488
-1.6	0.17781	0.12537	0.10395	0.09242	0.08525	0.08036	0.07682	0.07413	0.07203	0.07034	0.06895	0.06779	0.06680	0.06596	0.06522
-1.5	0.18717	0.13620	0.11529	0.10400	0.09695	0.09214	0.08865	0.08600	0.08393	0.08225	0.08088	0.07973	0.07875	0.07791	0.07718
-1.4	0.19743	0.14824	0.12800	0.11705	0.11020	0.10552	0.10212	0.09954	0.09751	0.09588	0.09454	0.09342	0.09246	0.09164	0.09093
-1.3	0.20871	0.16162	0.14223	0.13173	0.12515	0.12065	0.11738	0.11490	0.11295	0.11138	0.11009	0.10901	0.10809	0.10730	0.10661
-1.2	0.22114	0.17650	0.15813	0.14818	0.14195	0.13768	0.13459	0.13223	0.13039	0.12890	0.12767	0.12665	0.12578	0.12503	0.12437
-1.1	0.23485	0.19302	0.17584	0.16654	0.16073	0.15675	0.15386	0.15166	0.14994	0.14855	0.14741	0.14645	0.14564	0.14494	0.14434
-1.0	0.25000	0.21132	0.19550	0.18695	0.18161	0.17796	0.17531	0.17330	0.17172	0.17045	0.16940	0.16852	0.16778	0.16714	0.16659
-0.9	0.26674	0.23155	0.21723	0.20950	0.20469	0.20140	0.19901	0.19720	0.19578	0.19464	0.19370	0.19291	0.19224	0.19167	0.19117
-0.8	0.28522	0.25382	0.24110	0.23426	0.23001	0.22711	0.22500	0.22341	0.22216	0.22115	0.22032	0.21963	0.21904	0.21854	0.21810
-0.7	0.30560	0.27820	0.26716	0.26125	0.25757	0.25507	0.25326	0.25189	0.25081	0.24994	0.24923	0.24864	0.24813	0.24770	0.24732
-0.6	0.32798	0.30472	0.29540	0.29042	0.28733	0.28523	0.28371	0.28256	0.28165	0.28093	0.28033	0.27983	0.27941	0.27905	0.27873
-0.5	0.35242	0.33333	0.32572	0.32166	0.31915	0.31744	0.31620	0.31527	0.31454	0.31395	0.31346	0.31306	0.31272	0.31242	0.31217
-0.4	0.37888	0.36392	0.35797	0.35480	0.35284	0.35150	0.35054	0.34981	0.34924	0.34878	0.34841	0.34809	0.34783	0.34760	0.34740
-0.3	0.40723	0.39624	0.39188	0.38956	0.38812	0.38715	0.38645	0.38591	0.38550	0.38516	0.38489	0.38466	0.38446	0.38429	0.38415
-0.2	0.43717	0.42999	0.42714	0.42562	0.42468	0.42404	0.42358	0.42324	0.42296	0.42274	0.42257	0.42241	0.42229	0.42218	0.42208
-0.1	0.46827	0.46473	0.46333	0.46258	0.46212	0.46180	0.46157	0.46140	0.46127	0.46116	0.46107	0.46100	0.46094	0.46088	0.46083
0.0	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000

Table 4: The Left-tailed Probabilities $P(T < t)$ for negative t, with df=16–30

t	**df 16**	**df 17**	**df 18**	**df 19**	**df 20**	**df 21**	**df 22**	**df 23**	**df 24**	**df 25**	**df 26**	**df 27**	**df 28**	**df 29**	**df 30**
-4.0	0.00052	0.00046	0.00042	0.00038	0.00035	0.00032	0.00030	0.00028	0.00026	0.00025	0.00023	0.00022	0.00021	0.00020	0.00019
-3.9	0.00064	0.00058	0.00052	0.00048	0.00044	0.00041	0.00038	0.00036	0.00034	0.00032	0.00030	0.00029	0.00027	0.00026	0.00025
-3.8	0.00079	0.00072	0.00066	0.00060	0.00056	0.00052	0.00049	0.00046	0.00044	0.00041	0.00039	0.00037	0.00036	0.00034	0.00033
-3.7	0.00097	0.00089	0.00082	0.00076	0.00071	0.00066	0.00063	0.00059	0.00056	0.00053	0.00051	0.00049	0.00047	0.00045	0.00043
-3.6	0.00120	0.00110	0.00102	0.00095	0.00089	0.00084	0.00080	0.00076	0.00072	0.00069	0.00066	0.00063	0.00061	0.00059	0.00057
-3.5	0.00148	0.00137	0.00128	0.00120	0.00113	0.00107	0.00101	0.00096	0.00092	0.00088	0.00085	0.00082	0.00079	0.00076	0.00074
-3.4	0.00183	0.00170	0.00160	0.00150	0.00142	0.00135	0.00129	0.00123	0.00118	0.00113	0.00109	0.00106	0.00102	0.00099	0.00096
-3.3	0.00226	0.00212	0.00199	0.00188	0.00179	0.00170	0.00163	0.00157	0.00151	0.00145	0.00140	0.00136	0.00132	0.00128	0.00125
-3.2	0.00279	0.00262	0.00248	0.00236	0.00225	0.00215	0.00207	0.00199	0.00192	0.00186	0.00180	0.00175	0.00170	0.00166	0.00162
-3.1	0.00344	0.00325	0.00309	0.00295	0.00282	0.00271	0.00261	0.00252	0.00244	0.00237	0.00231	0.00225	0.00219	0.00214	0.00209
-3.0	0.00424	0.00403	0.00384	0.00368	0.00354	0.00341	0.00330	0.00320	0.00310	0.00302	0.00294	0.00287	0.00281	0.00275	0.00269
-2.9	0.00522	0.00498	0.00477	0.00459	0.00443	0.00428	0.00415	0.00404	0.00393	0.00383	0.00375	0.00367	0.00359	0.00352	0.00346
-2.8	0.00642	0.00615	0.00592	0.00571	0.00553	0.00536	0.00522	0.00509	0.00496	0.00485	0.00475	0.00466	0.00458	0.00450	0.00443
-2.7	0.00789	0.00759	0.00733	0.00709	0.00689	0.00670	0.00654	0.00639	0.00625	0.00613	0.00602	0.00591	0.00581	0.00573	0.00564
-2.6	0.00967	0.00934	0.00905	0.00879	0.00856	0.00836	0.00817	0.00800	0.00785	0.00771	0.00758	0.00747	0.00736	0.00726	0.00716
-2.5	0.01184	0.01147	0.01115	0.01087	0.01062	0.01039	0.01018	0.01000	0.00983	0.00967	0.00953	0.00940	0.00928	0.00916	0.00906
-2.4	0.01446	0.01406	0.01371	0.01340	0.01312	0.01287	0.01265	0.01244	0.01225	0.01208	0.01193	0.01178	0.01164	0.01152	0.01140
-2.3	0.01762	0.01719	0.01681	0.01648	0.01617	0.01590	0.01565	0.01543	0.01522	0.01504	0.01486	0.01470	0.01456	0.01442	0.01429
-2.2	0.02142	0.02096	0.02055	0.02019	0.01986	0.01957	0.01930	0.01906	0.01884	0.01863	0.01845	0.01827	0.01811	0.01796	0.01782
-2.1	0.02597	0.02548	0.02505	0.02466	0.02431	0.02399	0.02371	0.02345	0.02321	0.02299	0.02279	0.02260	0.02243	0.02227	0.02212
-2.0	0.03139	0.03087	0.03041	0.03000	0.02963	0.02930	0.02900	0.02872	0.02847	0.02824	0.02802	0.02783	0.02764	0.02747	0.02731
-1.9	0.03780	0.03727	0.03679	0.03636	0.03597	0.03563	0.03531	0.03502	0.03476	0.03451	0.03429	0.03408	0.03389	0.03371	0.03354
-1.8	0.04537	0.04482	0.04432	0.04388	0.04348	0.04312	0.04280	0.04250	0.04222	0.04197	0.04174	0.04152	0.04132	0.04114	0.04096
-1.7	0.05424	0.05368	0.05317	0.05272	0.05232	0.05195	0.05161	0.05131	0.05103	0.05077	0.05053	0.05031	0.05011	0.04992	0.04974
-1.6	0.06458	0.06401	0.06350	0.06305	0.06264	0.06227	0.06193	0.06162	0.06134	0.06108	0.06084	0.06062	0.06041	0.06022	0.06004
-1.5	0.07654	0.07598	0.07548	0.07502	0.07462	0.07425	0.07391	0.07361	0.07333	0.07307	0.07283	0.07261	0.07240	0.07221	0.07203
-1.4	0.09030	0.08975	0.08926	0.08882	0.08842	0.08806	0.08773	0.08743	0.08715	0.08690	0.08667	0.08645	0.08625	0.08606	0.08589
-1.3	0.10601	0.10548	0.10500	0.10458	0.10419	0.10384	0.10353	0.10324	0.10297	0.10273	0.10250	0.10230	0.10210	0.10192	0.10175
-1.2	0.12380	0.12330	0.12285	0.12244	0.12208	0.12175	0.12145	0.12118	0.12093	0.12069	0.12048	0.12028	0.12010	0.11993	0.11977
-1.1	0.14380	0.14333	0.14291	0.14254	0.14220	0.14189	0.14161	0.14136	0.14112	0.14091	0.14071	0.14052	0.14035	0.14019	0.14004
-1.0	0.16610	0.16567	0.16528	0.16494	0.16463	0.16435	0.16409	0.16386	0.16364	0.16345	0.16326	0.16309	0.16294	0.16279	0.16265
-0.9	0.19073	0.19035	0.19000	0.18969	0.18941	0.18916	0.18893	0.18872	0.18853	0.18835	0.18819	0.18804	0.18790	0.18777	0.18764
-0.8	0.21771	0.21737	0.21707	0.21680	0.21656	0.21633	0.21613	0.21595	0.21578	0.21562	0.21548	0.21535	0.21522	0.21511	0.21500
-0.7	0.24699	0.24670	0.24644	0.24621	0.24600	0.24580	0.24563	0.24547	0.24533	0.24520	0.24507	0.24496	0.24485	0.24475	0.24466
-0.6	0.27845	0.27821	0.27799	0.27780	0.27762	0.27746	0.27732	0.27719	0.27706	0.27695	0.27685	0.27675	0.27667	0.27658	0.27651
-0.5	0.31194	0.31174	0.31157	0.31141	0.31127	0.31114	0.31102	0.31091	0.31081	0.31072	0.31064	0.31056	0.31049	0.31042	0.31036
-0.4	0.34722	0.34707	0.34693	0.34681	0.34670	0.34660	0.34651	0.34642	0.34635	0.34628	0.34621	0.34615	0.34610	0.34604	0.34600
-0.3	0.38402	0.38391	0.38381	0.38372	0.38364	0.38356	0.38350	0.38344	0.38338	0.38333	0.38328	0.38324	0.38320	0.38316	0.38312
-0.2	0.42200	0.42193	0.42186	0.42180	0.42175	0.42170	0.42166	0.42162	0.42158	0.42155	0.42152	0.42149	0.42146	0.42144	0.42142
-0.1	0.46079	0.46076	0.46072	0.46070	0.46067	0.46065	0.46063	0.46061	0.46059	0.46057	0.46056	0.46054	0.46053	0.46052	0.46050
0.0	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000